

## NOTES ON DIMER MODEL

A matching  $M$  of the bipartite graph  $G$  gives a chain  $c \in C_1(G, \mathbb{Z})$  that has the properties  $c = \sum \epsilon_i e_i$  ( $\epsilon_i = 0$  or  $1$ ) and  $\partial c = \sum w - \sum b$ , where  $w$  represents white vertices, and  $b$  represents black vertices. If  $M'$  is another matching, and  $c'$  is the corresponding chain, then  $\partial(c' - c) = 0$ . If  $G$  is planar, then there must be a collection  $r$  of faces such that  $r = \sum \epsilon'_i f_i$  and  $\partial r = c' - c$ . The intersection number of  $c' - c$  with any 1-cycle of  $G^*$  (the dual of  $G$ ) is 0.

Fix a reference matching  $M_0$ , the corresponding  $c_0$ , and a face  $f_0$ , then for any face  $f$ , find a path in  $G^*$  from  $f_0$  to  $f$ , define the *height function*

$$h(f, M) = \text{intersection of } c - c_0 \text{ to } G.$$

If  $G$  is on the torus, the intersection of  $c - c_0$  with the longitude  $G_x$  and with the latitude  $G_y$  generate  $H_1$  of the torus. The height function is defined up to  $\mathbb{Z}^2$ . Horizontal change  $j$  and vertical change  $k$ .

The energy is a cochain  $\mathcal{E} \in C^1(G, \mathbb{R})$ . A Gibbs measure is a probability on  $\mathcal{M}(G)$  such that the probability of  $M \in \mathcal{M}(G)$  is proportional to  $e^{-\mathcal{E}(M)}$ .  $\mathbb{Z}^2$ -invariant and ergodic measures are called EGM. For any EGM  $\mu$ , let  $s = \mathbb{E}_\mu(j)$  and  $t = \mathbb{E}_\mu(k)$ . The pair  $(s, t)$  is the slop of  $\mu$ .

On  $\mathcal{M}(G_n)$ , define  $\mu_n(M) = e^{-\mathcal{E}(M)}/Z$ . For a fixed pair  $(s, t) \in \mathbb{R}^2$ , let  $\mathcal{M}_{s,t}(G_n)$  be the collection of matchings of  $G_n$  with height change  $([ns], [nt])$ . Use  $\mu_n(s, t)$  to denote the measure on  $\mathcal{M}_{s,t}(G_n)$  induced from  $\mu_n$ .

Gauge transformation. Energy function could be changed. If  $\mathcal{E}_1$  and  $\mathcal{E}_2$  are homologous, then  $\mathcal{E}_1(c) - \mathcal{E}_2(c) = d\alpha(c) = \alpha(\partial c) = \alpha(\sum w - \sum b)$ , independent of  $c$ . So  $e^{-\mathcal{E}_1(M)} = C e^{-\mathcal{E}_2(M)}$ , i.e. they determine the same Gibbs measure. The magnetic flux through a simple closed cycle  $c \in C_1(G)$  is  $\mathcal{E}(c)$ . The gauge equivalent classes of energies of  $G_1$  is  $C^1(G_1, \mathbb{R})/dC^0(G_1, \mathbb{R}) \cong dC^1(\mathbb{T}, \mathbb{R}) \oplus H^1(\mathbb{T}, \mathbb{R})$ .